QED in optically active media: Enhanced spontaneous emission and chiral yet parity conserving corrections to the electron mass

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Abstract

The electromagnetic field inside an isotropic optically active medium is quantized on the base of Fedorov's model for optical activity. The modified photon propagator is derived. Using this result it is shown that the QED correction to the electron mass contains chiral terms which nevertheless are parity invariant. The spontaneous emission rate of a two-level atom is shown to increase in the medium. The spontaneously emitted radiation is partially polarized.

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Introduction

In recent years there has been a growing attention to quantum electrodynamics (QED) in dielectric media [1, 2, 3, 4, 5]. This field is of interest for several reasons. One motivation is the study of squeezed states of light connected with the dielectric [3]. Another one is the more accurate description of Casimir forces between conducting plates which are special cases of dielectrics (see, e.g., Ref. [6] and references therein). Despite these efforts the case of optically active media, for which (in most models) the electric displacement vector depends also on derivatives of the electric field, was not often studied. To the author's knowledge there is only a work of Eimerl [7] in which the photon self energy is used to give predictions for the parameters describing the medium, and a paper of Woolley [8] where a nonrelativistic QED calculation for the molecular magnitudes leading to optical activity is presented.

The influence of the quantized electric field on quantum matter in the presence of an optically active medium seems not to be discussed in the literature. In this paper it is considered how mass renormalization for the electron and spontaneous emission will be modified by the medium. Compared to ordinary dielectrics new effects mainly arise because of the different behaviour of left and right circularly polarized field modes. Since these polarization modes are the eigenmodes of the spin operator many new effects depending on the spin and the angular momentum of atoms or electrons can be imagined. This is the case for the self energy of the electron, for instance, which will be shown to include chiral terms. The spontaneous emission of photons, which in the approximation made here does not include spin or angular momentum of the atom, leads to partially polarized light as will be demonstrated later. Beside the examples studied here the Casimir effect for optically active media is under investigation. It should be mentioned, however, that the effects produced by the medium are in general small since the change in the refractive index in optically active media is small compared to the change in ordinary dielectrics.

Quantization of the electromagnetic field

The starting point of the calculations are the Maxwell equations

$$\nabla \cdot \vec{D} = \rho \quad , \nabla \times \vec{H} - \dot{\vec{D}} = \vec{j}$$
$$\nabla \cdot \vec{B} = 0 \quad , \nabla \times \vec{E} + \dot{\vec{B}} = \vec{0} \tag{1}$$

together with Fedorov's model for optical activity [9]

$$\vec{D} = \varepsilon(\vec{E} + \beta \nabla \times \vec{E})$$
$$\vec{B} = \mu(\vec{H} + \beta \nabla \times \vec{H}) \tag{2}$$

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where only the case of an isotropic medium is considered. For the sake of simplicity ε and μ, which are not related to the optical activity, will be replaced in this paper by their vacuum value, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$. Fedorov's model was chosen to preserve gauge invariance (for a compilation of other models see Ref. [10]). The reason for this is that the nontrivial part of the material equations (2) depends on $\vec{E}$ and $\vec{B}$ only through their curl. This implies that only the transverse electromagnetic field, which is not affected by gauge transformations, will be modified by the medium.

Switching to the gauge potentials $\phi, \vec{A}$ and imposing the Lorentz gauge condition on them one easily deduces the field equations

$$
\begin{aligned}
\Box \phi &= 0 \\
\Box \vec{A} &= -\frac{\beta}{c^2}\partial_t^2\{2\nabla \times \vec{A} + \beta\nabla \times \nabla \times \vec{A}\}
\end{aligned}
\tag{3}
$$

in absence of free charges ρ or sources $\vec{j}$. The d'Alembertian $\Box$ is defined by $\partial_t^2/c^2 - \Delta$. To quantize these fields the method of normal mode decomposition will be used. To do so one first shows that the scalar products

$$
\begin{aligned}
(\phi, \psi) &:= \frac{-i\varepsilon_0}{\hbar c^2}\int d^3x\{\dot{\phi}^\dagger\psi - \phi^\dagger\dot{\psi}\} \\
(\vec{A}, \vec{A}') &:= \frac{-i\varepsilon_0}{\hbar}\int d^3x\Big\{\dot{\vec{A}}^\dagger \cdot \vec{A}' - \vec{A}^\dagger \cdot \dot{\vec{A}}' + 2\beta\Big[(\nabla \times \dot{\vec{A}}^\dagger) \cdot \vec{A}' - \vec{A}^\dagger \cdot (\nabla \times \dot{\vec{A}}')\Big] + \\
&\qquad \beta^2\Big[(\nabla \times \dot{\vec{A}}^\dagger) \cdot (\nabla \times \vec{A}') - (\nabla \times \vec{A}^\dagger) \cdot (\nabla \times \dot{\vec{A}}')\Big]\Big\}
\end{aligned}
\tag{4}
$$

are conserved for solutions ϕ, ψ and $\vec{A}, \vec{A}'$ of Eqs. (3). The general solution of the field equations can be found by Fourier transformation. It is given by

$$
\begin{aligned}
\phi(x^\mu) &= \sqrt{\frac{\hbar c}{(2\pi)^3\varepsilon_0}}\int \frac{d^3k}{\sqrt{2k}}\Big\{a_0(\vec{k})e^{-ik\cdot x} + h.c.\Big\} \\
\vec{A}(x^\mu) &= \sqrt{\frac{\hbar}{(2\pi)^3\varepsilon_0 c}}\int d^3k\Big\{\vec{e}_3(\vec{k})\frac{a_3(\vec{k})e^{-ik\cdot x}}{\sqrt{2k}} + \vec{e}_+(\vec{k})\frac{a_+(\vec{k})e^{-ik_+\cdot x}}{\sqrt{2k_+^0(1-\beta k)}} + \vec{e}_-(\vec{k})\frac{a_-(\vec{k})e^{-ik_-\cdot x}}{\sqrt{2k_-^0(1+\beta k)}} + h.c.\Big\}
\end{aligned}
\tag{5}
$$

Here $\vec{e}_i$ are the complex polarization vectors defined by $\vec{e}_3 := \vec{k}/k$ and $i\vec{e}_3 \times \vec{e}_\pm = \pm\vec{e}_\pm$. Although the calculation is not Lorentz invariant it is convenient to adopt the covariant notation $a \cdot b := a^\mu b_\mu = a^0 b^0 - \vec{a}\cdot\vec{b}$ and $x^0 := ct$. Throughout the paper the summation convention over repeated indices is adopted. Greek indices are running from 0 to 3 and latin ones from 1 to 3. The various four-wavevectors are defined by $k^\mu := (k, \vec{k})$ and $k_\pm^\mu := (k_\pm^0, \vec{k})$ where

$$
k_\pm^0 := \frac{k}{\sqrt{1 \pm \beta k - \beta^2 k^2}} \, .
\tag{6}
$$

Here one can see the main effect of the presence of the optically active medium. The dispersion relation between frequency and wavevector of the left and right circularly polarized modes ($\pm$) has been changed from $k^0 = k$ to $k^0 = k_\pm^0$. Since $k_\pm^0$ diverges for plus-modes with $\beta k = 1 + \sqrt{2}$ and minus-modes with $\beta k = \sqrt{2} - 1$ it is clear that the theory describes physics only for electromagnetic waves with $\beta k \ll 1$. This should not be regarded as a shortcoming since in real media optical activity appears only well below the high energy physics regime. The introduction of a cutoff in momentum space is therefore physically meaningful.

It should be mentioned that the dispersion relation $k^0 = k_\pm^0$ is a good description only for a certain part of the low energy frequency range. This is a consequence of describing optical activity within a local field theory. Despite this fact local theories can still be a useful tool estimate the order of magnitude of the medium's influence on quantum matter.

In Eq. (5) it also becomes apparent that only the transverse field modes ($\pm$) feel the presence of the medium in Fedorov's model. Hence no gauge transformation can alter the effect of the medium on the Maxwell field.

The quantization of the Maxwell field is now an easy task. One demands the usual Lorentz gauge commutation relations

$$[a_\alpha(\vec{k}), a_\beta^\dagger(\vec{k}')] = \delta(\vec{k} - \vec{k}')\delta_{\alpha\beta} \tag{7}$$

with $\alpha, \beta = 0, +, -, 3$ and $-\delta_{00} = \delta_{33} = \delta_{++} = \delta_{--} = 1$ as well as $\delta_{\alpha\beta} = 0$ for any other combination of α and β. In addition one has to impose the Gupta-Bleuler condition $\partial_\mu A^{\mu(+)}|\psi\rangle = 0$ on the physical states $|\psi\rangle$ (see, e.g., Ref. [11]).

The self energy of the electron

To calculate the first order mass correction for the electron the Feynman propagator

$$D^{\mu\nu}(x - y) = -i\theta(x^0 - y^0)[A^{\mu(+)}(x), A^{\nu(-)}(y)] - i\theta(y^0 - x^0)[A^{\nu(+)}(y), A^{\mu(-)}(x)] \tag{8}$$

of the photon field is needed. From Eq. (5) it is clear that only the spatial part of the propagator is affected by the medium. It is useful to rewrite D^{ij} into the form $D_{vac}^{ij} + D_+^{ij} + D_-^{ij}$ where D_{vac}^{ij} stands for the vacuum contribution and $D_\pm^{ij}$ describes the changes of the propagation of the left and right circularly polarized modes induced by the medium. Using standard methods (including an integration over k^0 with the residue formula and the use of $\vec{e}_+^i(-\vec{k})\vec{e}_+^{j*}(-\vec{k}) = \vec{e}_+^{i*}(\vec{k})\vec{e}_+^j(\vec{k})$ and a similar equation for $\vec{e}_-$) the expression for $D_\pm^{ij}$ in momentum space is found to be

$$D_\pm^{ij}(k^\mu) = \frac{\hbar}{\varepsilon_0 c}\vec{e}_\pm^i(\vec{k})\vec{e}_\pm^{j*}(\vec{k})\left\{\frac{1}{(1 \mp \beta k)^2[(k^0)^2 - (k_\pm^0)^2 + i\varepsilon]} - \frac{1}{(k^0)^2 - k^2 + i\varepsilon}\right\}. \tag{9}$$

Having found the photon propagator one is ready to calculate the the first order on-shell mass correction [11]

$$\delta m = \frac{ie^2}{(2\pi)^4\hbar c}\int d^4k D^{\alpha\beta}(k)\gamma_\alpha S_F(q - k)\gamma_\beta \tag{10}$$

with $S_F(q) = [\gamma^\mu q_\mu - \mu + i\varepsilon]^{-1}$ being the Fermion Propagator. $\hbar q^\mu$ is the external momentum of the electron and $\mu := mc/\hbar$ is its Compton wavevector. If $\gamma^\mu q_\mu$ stands on the right of δm it can be replaced by μ since the external electron spinor ψ fulfills the free Dirac equation $(\gamma^\mu q_\mu - \mu)\psi = 0$. Again it is convenient to write δm in the form $\delta m_{vac} + \delta m_+ + \delta m_-$ where every term corresponds to the respective part of the photon propagator. Then $\delta m_\pm$ can be calculated in a standard manner by making a Feynman parametrization and performing the k^0 integral with the residue formula. Using $a^\mu\gamma_\mu\gamma_j = -\gamma_j a^\mu\gamma_\mu + 2a_j$ and $-(i/2k)\varepsilon_{ijl}\gamma^i\gamma^j k^l k^\alpha\gamma_\alpha = -k^0\gamma_5\gamma_l k^l/k + k\gamma_0\gamma_5$ as well as

$$\vec{e}_\pm^i\vec{e}_\pm^{j*} = \frac{1}{2}\left\{\delta^{ij} - \frac{1}{k^2}k^i k^j \mp \varepsilon^{ijl}\frac{k_l}{k}\right\} \tag{11}$$

($\varepsilon^{123} = -1$) one arrives at

$$\begin{aligned}
\delta m_\pm &= \frac{e^2}{(2\pi)^3 4\varepsilon_0 c^2}\int_0^1 dz \int d^3k\left\{\frac{1}{w^3} - \frac{1}{(1 \mp \beta k)^2 w_\pm^3}\right\} \times \\
&\quad \left\{\pm \gamma_0\gamma_5 k + \mp zq^0\gamma_5\gamma_l\frac{k^l}{k} - (q^i + k^i)\gamma_i - zq^0\gamma_0 + \frac{k^i}{k^2}\gamma_i(\vec{k}\cdot\vec{q}) \mp i\varepsilon^{ijl}\gamma_i q_j\frac{k_l}{k}\right\}
\end{aligned} \tag{12}$$

with $w_\pm := \sqrt{z^2\mu^2 + (\vec{k} - z\vec{q})^2 + (1 - z)((k_\pm^0)^2 - k^2)}$ and where w is defined by the same expression if $k_\pm^0$ is set equal to k. After fixing the direction of $\vec{q}$ to be parallel to the 3-axis the angular part of the integration can be performed as usual. In the remaining expression a high-frequency cutoff κ is introduced for the dk integral. As explained above this cutoff is needed since optical activity is only present in the low-energy regime. Since κ has to be chosen so that $\beta\kappa \ll 1$ is fulfilled it is sufficient to calculate δm only to first order in β. In this case the remaining integrations over z and k lead to closed expressions for δm which contain various polynomials and logarithms. For all allowed external momenta ($q\beta \ll 1$) the lowest order Taylor expansion

$$\delta m_+ + \delta m_- = \frac{e^2}{9\pi^2\varepsilon_0 c^2}\frac{\beta\kappa^3}{\mu^2}\left\{-3\mu\gamma_0\gamma_5 + q\gamma_5\gamma_3\right\} + O(\beta^2) + O(q^3) + O(\kappa^4) \tag{13}$$

gives an excellent approximation to these functions.

This result deserves some comments. First, it is obvious that δm does not only depend on $q^\mu q_\mu$ as in the vacuum but also on the three-vector $\vec{q}$. This implies that for on-shell amplitudes with $q^\mu q_\mu = \mu^2$ the propagation depends on the momentum of the electron in a non-trivial way. The reason is that the rest frame of the medium introduces a new time-like vector ζ^μ so that δm can depend on both $q^\mu q_\mu$ and $q^\mu \zeta_\mu$.

The second comment to Eq. (13) concerns the fact that $\delta m_+ + \delta m_-$ depends on the spin of the electron only through the chiral operators $\gamma_0 \gamma_5$ and $\gamma_5 \gamma_i$ which are known to be a pseudo-scalar and pseudo-vector, respectively. Nevertheless, δm behaves as a scalar under a parity transformation if one takes into account the transformation properties of the medium. For Fedorov's model the parameter β is a pseudo-scalar [9, 12] so that δm indeed has the right transformation properties.

The physical origin of the chiral mass corrections is hidden in the properties of the medium. As explained in Refs. [13, 14] the self energy of a particle is a consequence of the radiation reaction and not of the vacuum fluctuations of the electromagnetic field. Hence it is produced by the field emanated from the electron that interacts with the medium and turns back to the electron. The chiral nature of the self energy is caused by the different interaction of the medium with left and right circularly polarized light. This implies that field modes with different total angular momentum propagate differently (as can be seen in Eq. (9)) and the back reaction on the electron depends on its total angular momentum.

To estimate the magnitude of $\delta m_+ + \delta m_-$ a specific value for β is needed. This can be done by applying the method of Ref. [10] to Fedorov's model. The result is that β is given by $\lambda(n_+ - n_-)/(4\pi)$ where λ is the wavelegth of the light beam and $n_\pm$ are the refractive indices for left and right circularly polarized light. Adopting the values for quartz (that, however, is not an *isotropic* optically active medium) at a wavelength of 762 nm one can infer that β is given by 3.6×10^{-12} m. Then a very large value for κ is, e.g., $\kappa = 10^{11}$ m^{-1}. Inserting these values into Eq. (13) gives $(\delta m_+ + \delta m_-)/m \approx 10^{-6}$, the effect of the medium on the electron mass is very small.

Spontaneous emission of a two-level atom

Spontaneous emission of a photon during an atomic transition and the exponential decay of the excitation probability arise from the destructive interference between the transition amplitudes to all photon states. The calculation of the decay factor for a two-level atom with excited state $|e\rangle$ and ground state $|g\rangle$ is a standard task in quantum optics and can be found in many textbooks (see, eg. Ref. [6]). In the Markov approach the Heisenberg equations of motion of the atomic density matrix and the annihilation operators of the radiation field is solved. In this paper the Hamiltonian

$$H = H_A + H_{e.m.} - \vec{d} \cdot \vec{E}^\perp (|e\rangle\langle g| + |g\rangle\langle e|) \tag{14}$$

is used, where $H_A := E_e |e\rangle\langle e| + E_g |g\rangle\langle g|$ contains the internal energy of the atom and $H_{e.m}$ describes the time evolution of the free radiation field. $\vec{d} := \langle e|q\vec{x}|g\rangle$ is the dipole moment of the atomic transition and $\vec{E}^\perp = -\partial_t \vec{A}^\perp$ is the transversal part of the electric field. In Fedorov's model $\vec{E}^\perp$ can be derived from Eq. (5) and the free evolution of the Maxwell field is governed by the Hamiltonian

$$H_{e.m.} = \int d^3k \sum_{\lambda=\pm} \hbar c k_\lambda^0 a_\lambda^\dagger(\vec{k}) a_\lambda(\vec{k}) \tag{15}$$

where the ground state energy was removed. Since the atom couples only to the transversal part of the radiation field the longitudinal and scalar part was neglected in $H_{e.m.}$.

The essential step of the Markov approach is the Markovian approximation in the formal solution for the annihilation operators (see, e.g., Ref. [6]). Following these steps the decay rate of the excited state is found to be

$$\gamma = 2\pi \int d^3k \sum_{\lambda=\pm} |C_\lambda(\vec{k})|^2 \delta(c k_\lambda^0 - \omega_0) . \tag{16}$$

Here $\omega_0 := (E_e - E_g)/\hbar$ is the atomic transition frequency and $C_\lambda(\vec{k})$ are the expansion coefficients of the electric field operator defined by $\vec{d} \cdot \vec{E} =: \hbar \int d^3k \sum_{\lambda=\pm} (i C_\lambda(\vec{k}) a_\lambda(\vec{k}) + h.c.)$. The δ distribution guarantees energy conservation. It also implies that the Fedorov model as well as any other local theory of optical activity can predict the spontaneous emission rate much better than the mass correction since only field modes with a certain frequency can contribute to it. The variation of the parameter β with the field frequency, which can

only be included in nonlocal theories, does therefore not alter the results. After evaluation of the integrals in Eq. (16) the contribution $\gamma_\pm$ of the left and right circularly polarized modes turn out to be

$$\gamma_\pm = \frac{\gamma_0}{2} \frac{\hat{k}_\pm^5}{k_0^5 (1 \mp \beta\hat{k}_\pm)^2 (1 \pm \beta\hat{k}_\pm)} \tag{17}$$

with $k_0 := \omega_0/c$. The wavevector belonging to modes with energy $\hbar\omega_0$ is given by $\hat{k}_\pm := k_0(\sqrt{1 + 2\beta^2 k_0^2} \pm \beta k_0)/(1 + \beta^2 k_0^2)$. The decay rate in free space is denoted by $\gamma_0 := \vec{d}^2 k_0^3/(3\pi\hbar\varepsilon_0)$. To a very good approximation the total decay rate $\gamma = \gamma_+ + \gamma_-$ is then given by

$$\gamma = \gamma_0(1 + 18\beta^2 k_0^2) + O(\beta^4 k_0^4) . \tag{18}$$

This result shows that an isotropic optically active medium always increases the spontaneous emission rate. This is due to the fact that the decay factor, being the inverse of the lifetime, has to transform as a scalar under parity transformations. Hence only even powers of β can occur in γ. That the coefficient of β^2 is positive can be understood by looking at $\gamma_\pm$ as a function of $\hat{k}_\pm$. Since $\hat{k}_\pm = k_0(1 \pm \beta k_0) + O(\beta^3 k_0^3)$ we need only to consider the lowest order term in $\hat{k}_\pm$ depending on β. Then $\hat{k}_+$ and $\hat{k}_-$ are shifted by the same amount from their vacuum value, but with opposite sign. As is well known Eq. (16) states that most of the k-dependence of γ is caused by the density of the field modes (see, e.g., Ref. [6]). This density increases more if k is shifted to a higher value ($\hat{k}_+$) than it decreases if k is shifted by the same amount to a lower value ($\hat{k}_-$). Thus the grow of γ_+ is larger than the decrease of γ_- so that the total decay rate increases. If β is less than zero γ_- is larger than γ_+. This argument also shows that the spontaneously emitted radiation is partially polarized since for $\beta > 0$, say, more left circularly than right circularly polarized photons are produced.

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